Pergamon

Int. J. Solids Structures Vol. 33, No. 3, pp. 301-314, 1996
Elsevier Science Ltd
Printed in Great Britain
0020-7683/96 $9.50 + .00

0020-7683(95)00037-2

TRANSIENT DEFORMATION MODES OF SQUARE PLATES SUBJECTED TO EXPLOSIVE LOADINGS

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(*Received* 15 *November* 1994; *in revised form* 6 *February* 1995)

Abstract—The phenomenon of transient deformation of square plates under intense dynamic loading has not been studied fully, either in experiment or theory. In this paper the experimental method for obtaining transient deformation profiles of square plate under explosive loading is demonstrated and evaluated. An optical technique proved to be reliable and accurate in obtaining the transient deformation mode of the square plate. Elastoplastic numerical analysis which includes finite deflection, material elasticity and strain hardening was carried out for one of the tests on an aluminium alloy plate to reproduce the transient response of the plate under measured pressure. The transient deformation profiles have also been obtained successfully by the numerical method and show good agreement with the experimental results. Numerical results are also presented for the impulsively loaded rectangular plates tested by Jones *et al.* [Jones, N., Vran, T. and Tekin, S. A. (1970). *Int. J. Solids Structures* **6**, 1499–1512]. The combined experimental and numerical investigations provide valuable information for the understanding of transient behaviour of thin plates under explosive loading and for the development of an approximate procedure using the Rigid Plastic Method.

1. INTRODUCTION

Experimental work on square and rectangular plates subjected to blast loading has been published quite extensively (Jones *et al.*, 1970, 1971; Jones and Baeder, 1972; Huang *et al.* 1985; Nurick, 1985; Huang *et al.* 1986; Nurick *et al.* 1986; Olson *et al.* 1993), but very few have touched the topic of the transient deformation mode.

Kiyota and Fujita (1963) and Hudson (1951) studied the deformation process of clamped circular plates under impulsive loading and the silhouette high speed photographs of a thin lead plate by Kiyota and Fujita (1963) showed the emergence of a frustum shape leading to a cone. The transient deformation mode of a circular plate was summarized by Johnson (1972) and it was indicated that for a clamped circular plate under initial velocity the circular central portion of the plate maintains its given transverse speed until overtaken by an inward travelling bending wave. The deformation mode of a clamped rectangular copper plate under impulsive loading was investigated by Duncan (1968) and a striking difference of the permanent deformation profile between impulsive and hydrostatic loading was noted. The vertical displacement measurement showed a flat plateau which was contracting in area with time.

In work in which the present author was involved, a new experimental method was developed to record the deformation process of square plate under blast loading (Huang *et al.*, 1986). This is a significant improvement on the experimental method for obtaining the transient mode of a plate. For the examples presented in the above paper, no detailed information were given about pressure pulse and material properties, which are essential for theoretical analysis. There was a small error in the data processing and graph plotting which gave rise to asymmetric deformation profile about the plate centre in Fig. 5 of the paper (Huang *et al.*, 1986) which is corrected here.

Much theoretical work has also been reported on the dynamic inelastic response of rectangular and square plates subjected to blast loadings and literature reviews can be

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found in Jones (1989), and Nurick and Martin (1989). Cox and Morland (1959) studied the dynamic plastic response of a simply supported square plate and gave an exact solution for infinitesimal deflections. A mode approximate method was proposed by Martin and Symonds (1966) for impulsively loaded rigid plastic structures. The initial velocity field was obtained by minimizing the difference between the given initial velocity and that of the mode solution in a mean square sense. Nurick *et al.* (1987) used the mode approximation techniques in which the mode shape was updated during each time increment. Approximate theoretical procedures, which retain the influence of finite deflections, were developed by Jones (1971) and Baker (1975) for dynamically loaded plates. The method of Jones (1971) has proved to be powerful and has been used widely in the estimate of permanent deflection.

The previous theoretical work was mainly concerned with the prediction of maximum permanent deflection. In obtaining the maximum permanent deflection using the Rigid Plastic Method, a time-independent collapse deformation mode is generally adopted. Even though it can give quite good prediction of the maximum permanent deflection, the deformation mode used may be different from the actual transient mode in the case of intense dynamic loading. Yu and Chen (1992) investigated the dynamic rigid-plastic response of a rectangular plate at large deflections in which the transient phase was included. The final deflections of a plate subjected to impulsive loading were calculated and compared with other theoretical solutions as well as the experimental results of Jones *et al.* (1970).

A literature review indicated the topic of transient deformation mode has not been studied fully, neither experimentally nor theoretically. This is particularly the case for blast-loaded square and rectangular plates. In this paper the experimental method for obtaining transient deformation profiles of a square plate under blast loading is demonstrated and evaluated. Elastoplastic numerical analysis is carried out for one of the tests on an aluminium alloy plate to reproduce the transient response of the plate under measured pressure. Numerical results are also presented for the impulsively loaded rectangular plates tested by Jones *et al.* (1970).

2. NUMERICAL ANALYSIS

A numerical model of elastoplastic response of a clamped rectangular plate under rigid wedge impact (Zhu and Faulkner, 1991, 1994) and pressure pulses (Zhu, 1990) was developed using the Variational Finite Difference Method (VFDM), which includes the influence of finite transverse deformation and material strain hardening. This numerical analysis was based on a minimum principle in dynamic plasticity (Lee and Ni, 1973). This assumes that the true acceleration field, $\ddot{U}_k = D^2U_k/DT^2$, of the body which has a known or predetermined displacement and velocity field at time T, is distinguished from all kinematically admissible ones by having the minimum value of the following functional

$$I = \int_{\bar{V}_0} s^{ij}\ddot{E}_{ij}\,d\bar{V}_0 + \frac{1}{2}\int_{\bar{V}_0} \rho_0 \ddot{U}^k \ddot{U}_k\,d\bar{V}_0 - \int_{S_T} T^k \ddot{U}_k\,dS - \int_{\bar{V}_0} \rho_0 F^k \ddot{U}_k\,d\bar{V}_0 \tag{1}$$

in which ρ_0 is the initial mass density.

The Lagrangian strain tensor is defined in the convected curvilinear coordinate system X_i

$$E_{ij} = \tfrac{1}{2}(U_{j;i} + U_{i;j} + U_{k;i}U^k_{;j}). \tag{2}$$

Here, $(\ )_{;i}$ denotes a covariant derivative of a variable with respect to X_i and the repetition of an index in a term indicates summation.

U_X, U_Y and U_Z are the corresponding physical components of the displacement vector of any particle in the plate, which can be expressed in terms of the corresponding displacements and their derivatives at the mid-plane by utilizing the Kirchhoff assumption

$$U_X = U - ZW_{,X}$$
$$U_Y = V - ZW_{,Y}$$
$$U_Z = W. \tag{3}$$

where U, V and W are the displacement components of a point at the mid-plane along the X, Y and Z axis, respectively.

For finite deformations, the equation of equilibrium on the surface (boundary condition) takes the form

$$T_k = (g_{ki} + U_{k,i})\nu_j s^{ij} \quad \text{on } S_T, \tag{4}$$

where g_{ki} is the metric tensor.

The lagrangian surface traction for the initial configuration and the pressure p at the deformed configuration can be expressed as

$$T_i \, \mathrm{d}S = \mathrm{p}\delta_i^k n_k \, \mathrm{d}a \tag{5}$$

where dS is the initial area element, da the corresponding area element after deformation, δ_i^k the Kronecker symbol and n_k the covariant unit normal to da.

For the rectangular plate under uniformly distributed pressure p but no body force, the functional I in eqn (1), may be specified as

$$I = \frac{E^2 L}{\rho_0}\left\{\int_v \frac{1}{E}(s_{xx}\ddot{E}_{xx} + 2s_{xy}\ddot{E}_{xy} + s_{yy}\ddot{E}_{yy})\,\mathrm{d}v + \frac{1}{2}\int_v \left[\left(\ddot{u} - z\frac{\partial \ddot{w}}{\partial x}\right)^2 + \left(\ddot{v} - z\frac{\partial \ddot{w}}{\partial y}\right)^2 + \ddot{w}^2\right]\mathrm{d}v \right.$$
$$\left. - \int_{s_t} \frac{p}{E}\left[\frac{\partial w}{\partial x}\left(\ddot{u} - z\frac{\partial \ddot{w}}{\partial x}\right) + \frac{\partial w}{\partial y}\left(\ddot{v} - z\frac{\partial \ddot{w}}{\partial y}\right) + \left(1 + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} - z\frac{\partial^2 w}{\partial x^2} - z\frac{\partial^2 w}{\partial y^2}\right)\ddot{w}\right]\mathrm{d}s\right\}, \tag{6}$$

where v and s_t are the non-dimensional volume and area, respectively; s_{xx}, s_{xy} and s_{yy} are the components of the Kirchhoff stress tensor which may be expressed in terms of the displacements and their rates by the constitutive relationships.

An elastic, perfectly strain hardening (bilinear) stress-strain relation was used in the numerical analysis. The material obeys the von Mises yield criterion. After discretization of every term in functional I [eqn (6)] is replaced by discrete values through a central finite difference scheme. The functional I may be replaced by a finite summation by using the 'trapezoidal rule' for integration. With the calculus of variations, the expressions for the accelerations at every nodal point at any time step, $t = q\Delta t$, may be obtained by minimizing the functional, I, with respect to the discrete accelerations as

$$\frac{\partial I^q}{\partial \ddot{u}_{ij}^q} = 0; \quad \frac{\partial I^q}{\partial \ddot{v}_{ij}^q} = 0; \quad \frac{\partial I^q}{\partial \ddot{w}_{ij}^q} = 0. \tag{7}$$

We then have the expression for $\ddot{u}_{ij}^q$, $\ddot{v}_{ij}^q$ and $\ddot{w}_{ij}^q$ by solving eqn (7). The displacement at time $t + \Delta t$ can be obtained by the central difference approximation

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_{ij}^{q+1} = \begin{Bmatrix} \ddot{u} \\ \ddot{v} \\ \ddot{w} \end{Bmatrix}_{ij}^{q} (\Delta t)^2 + 2\begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_{ij}^{q} - \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}_{ij}^{q-1}. \tag{8}$$

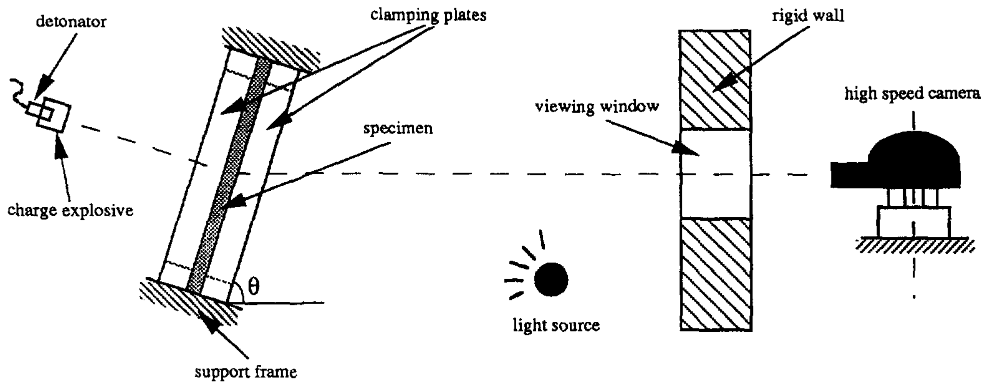


Fig. 1. Schematic drawing of experimental arrangement.

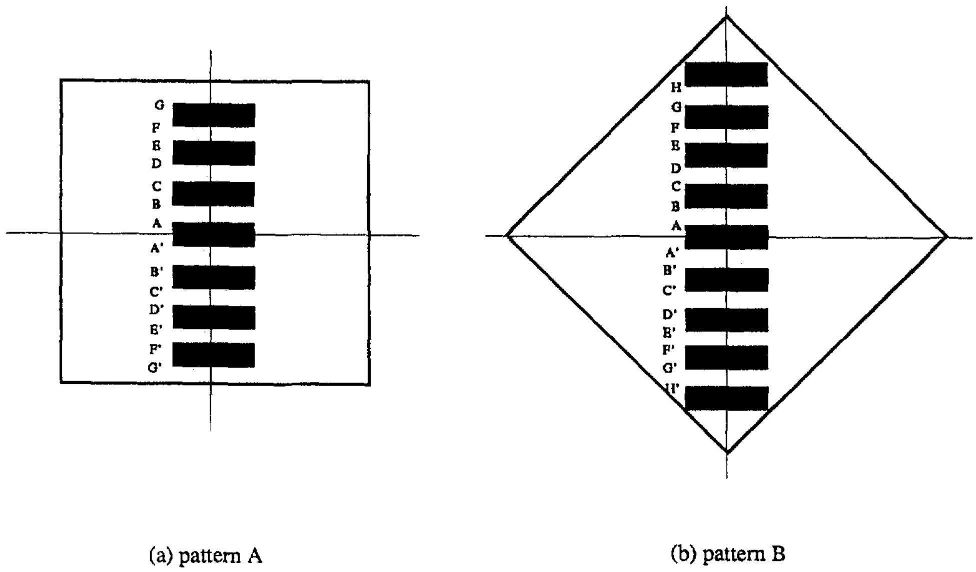


Fig. 2. Painting of black-white strips on surface of specimen.

3. EXPERIMENTAL METHOD

As mentioned in the Introduction, a new technique to measure the transient mode of square plate was reported by Huang *et al.* (1986). In this section the principle of our method will be briefly outlined. The experimental results for both aluminium alloy and mild steel plates will be examined and the measurements are processed by the present computer program to demonstrate and evaluate this experimental technique.

The experimental set-up is shown schematically in Fig. 1. The specimen was clamped and fixed at an inclined angle of θ to horizontal direction. A high speed streak camera was used to record the scanning line of specified nodes of the plate.

A specimen plate was painted with black and white stripes either along the symmetry line parallel to edge (pattern A) or along the diagonal (pattern B) of the plate (Fig. 2). More than 20 specimens were tested, which were made from either aluminium alloy or mild steel. The explosive charge was shaped in order to achieve a uniform pressure over the plate surface and this was checked with a number of pressure transducers attached on a plate surface and at the supporting edges. During the test, only pressures at the supporting edges of the plate were measured.

The trajectory of the lagrangian nodal pairs can be obtained according to the symmetry condition. Assume a pair of nodes, say A and A′ on plate specimen (Fig. 3), which are symmetric about the plate centre and have the corresponding image nodes a and a′. At

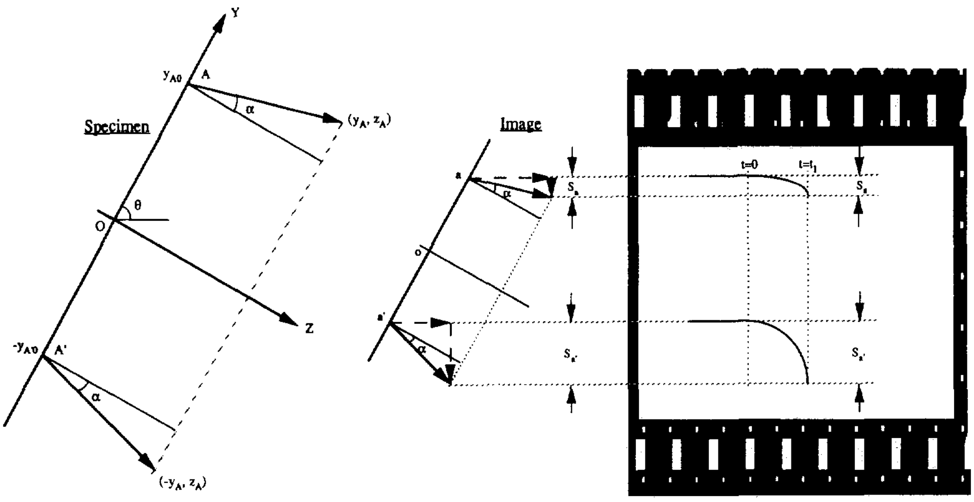


Fig. 3. Principle of optical method for obtaining transient deformation profiles of plate under blast loadings.

time t_1, nodes a and a′ have the same displacement S, and the component S_a and $S_{a'}$ can be read from the film as depicted in Fig. 3:

$$S_a = S\cos(\theta+\alpha) \tag{8a}$$

$$S_{a'} = S\cos(\theta-\alpha), \tag{9}$$

where α is the angle between the displacement vector and the normal to the plate. If we assume $K = S_a/S_{a'}$, then

$$\alpha = \arctan\left(\frac{1-k}{1+k}\cot\theta\right) \tag{10}$$

$$S = \frac{S_{a'}[(1+k)^2-4k\cos^2\theta]^{1/2}}{\sin 2\theta}. \tag{11}$$

If the objective to image ratio is ξ, the displacement components of the lagrangian nodes A and A′ at time t_1 can be determined in the $y-z$ coordinate system.

$$y_A = y_{A0}+\xi S\sin\alpha \tag{12}$$

$$z_A = \xi S\cos\alpha \tag{13}$$

and the symmetry condition yields:

$$y_{A'} = -y_A \tag{14}$$

$$z_{A'} = z_A. \tag{15}$$

y_{A0} is the y-coordinate of grid A at time $t = 0$.

The success of this technique is dependent on the symmetric deformation of the plate, therefore, only the tested specimens with symmetric permanent deformation profiles were selected for transient mode analysis. Good control of the light source is also important for obtaining high quality scanning records.

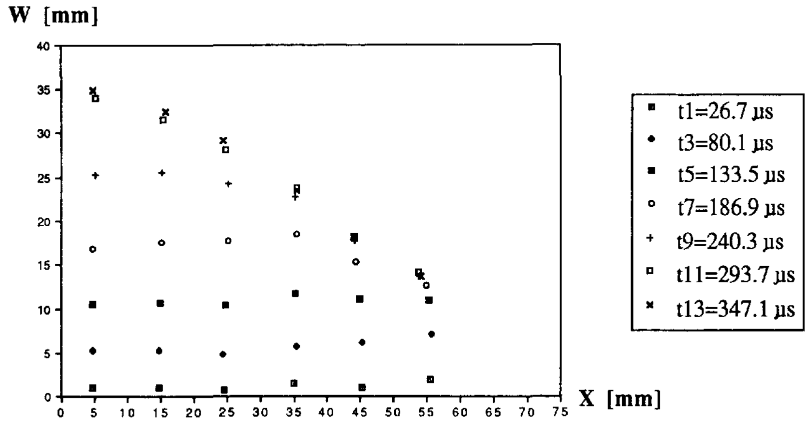


Fig. 4. Transient deformation profiles of aluminium alloy square plate (experimental method).

The value of S_a and $S_{a'}$ were read from the recorded film at 5 mm steps under a microscope and the equivalent time interval is 26.7 μs. Using a data processing and plotting program the transient deformation mode of the plate can be obtained.

Example 1 : *aluminium alloy plate* (*Pattern A*)

Transient deformation profiles of an aluminium alloy plate obtained using the above optical technique are shown in Fig. 4. The size of the plate was 150 × 150 × 1.0 (mm) with yield stress $\sigma_s = 85$ MPa. The recorded pressure time history can be approximated as an exponentially decaying pressure pulse (see Fig. 8) with peak pressure $p_0 = 4.8$ MPa and time constant $\tau = 85$ μs. Six lagrangian nodes along the symmetry line of the square plate ($X = 5$, 15, 25, 35, 45 and 55 mm, respectively) were traced. It was found that a flat plateau contracts toward the plate centre eventually, to form a more-or-less 'pyramid' shaped permanent deformation profile. For a given node, say $X = 45$ mm, it reaches a stationary state at time $t = 240$ μs or so. This can also be seen from the deflection-time history plot shown in Fig. 5(e). The deflection–time history for these six nodes are given in Fig. 5, in which the sequence of reaching the stationary state is shown from the edge to the plate centre. It transpires from all these experimental pictures in Figs 4 and 5 that edge lines of the square flat plateau shift toward the plate centre.

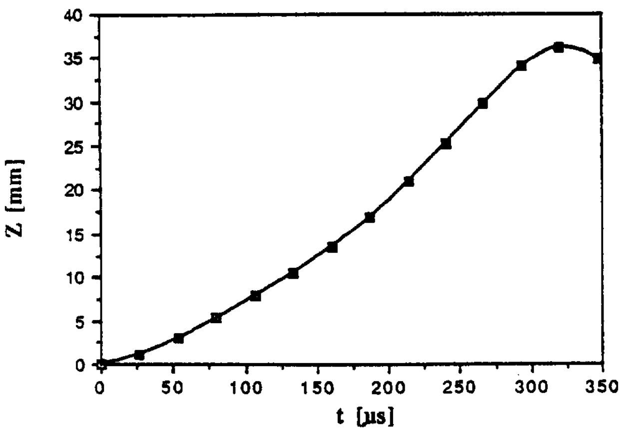
The permanent deflection of the aluminium alloy plate near the plate centre obtained by the optical technique was 34.85 mm, which is close to the dial gauge measured value of 36 mm.

Example 2 : *mild steel* (*Pattern A*)

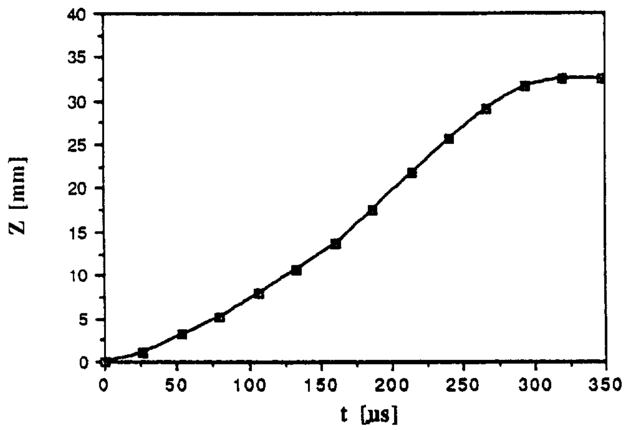
Another example is a tested case of mild steel plate with size 150 × 150 × 1.5 (mm) and yield stress $\sigma_s = 193$ MPa. Figure 6 gives the transient deformation profiles at various time instants. The maximum deflection is 24.63 mm and the maximum permanent deflection is 21.3 mm. The difference is due to the effect of material elasticity.

For all the selected specimens, the transient deformation modes were examined using the optical technique and the maximum permanent deflections were checked against the value from dial gauge measurements, which showed good agreement. This experimental method together with the present data processing program proved to be accurate and reliable. It should be stressed that as this optical method is based on the symmetry deformation condition, it can certainly be used for deformation mode study of rectangular plates and circular plates.

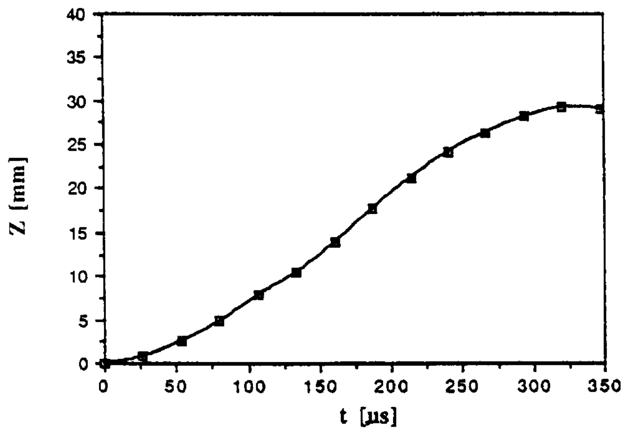
Since the specimen plate was clamped between two thick steel plates it was not possible to trace nodes near the boundary. The number of node pairs is six and eight for patterns A and B (see Fig. 2), respectively.



(a) Deflection history at X=5 mm

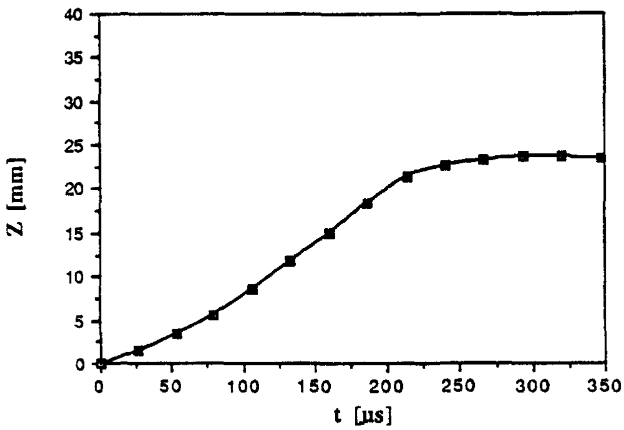


(b) Deflection history at X=15 mm

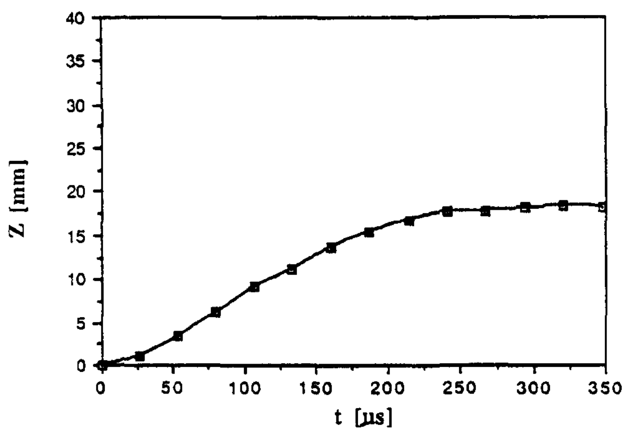


(c) Deflection history at X=25 mm

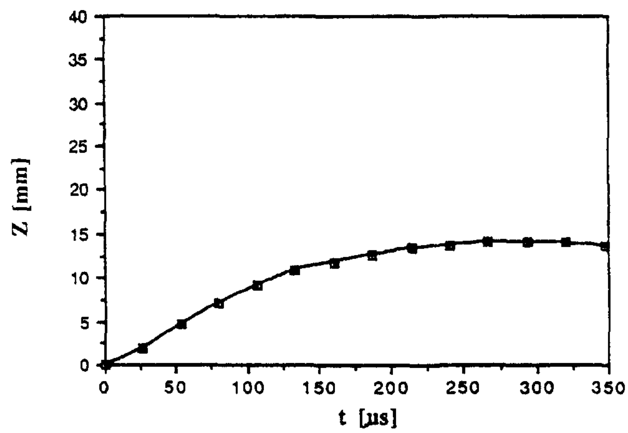
Fig. 5. Deflection–time histories at different positions along the symmetry line of aluminium alloy square plate (experimental method). (*Continued overleaf.*)



(d) Deflection history at X=35 mm



(e) Deflection history at X=45 mm



(f) Deflection history at X=55 mm

Fig. 5. *Continued.*

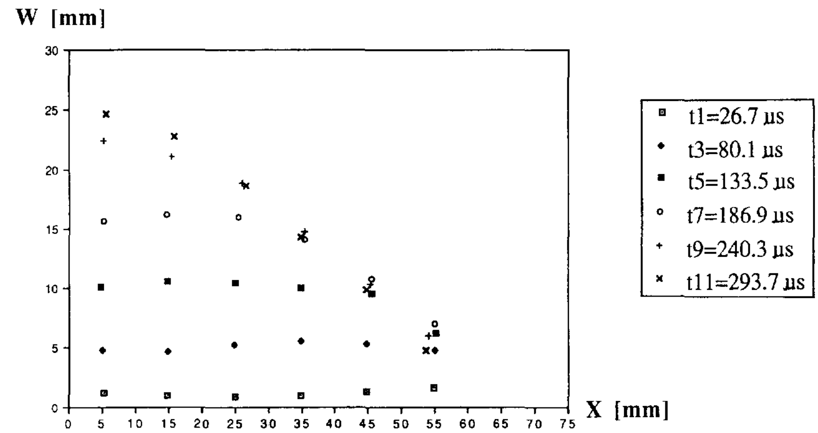


Fig. 6. Transient deformation profiles of mild steel square plate (experimental method).

4. NUMERICAL RESULTS AND DISCUSSION

4.1. *Maximum permanent deflections*

To validate the developed program which embodies the foregoing theoretical formulation, the iterative time-simulation technique has been checked against experimental data of Jones *et al.* (1970) in which the rectangular plates were loaded impulsively with an initial velocity.

For all the 19 cases of aluminium plates listed in Table 2 of Jones' paper, numerical calculations were performed (see Fig. 7a) and surprisingly good agreement has been found for the permanent deflection of aluminium alloy plates.

Numerical prediction and experimental results are shown in Fig. 7(b) for 22 hot-rolled mild steel specimens listed in the Table 1 of Jones *et al.* (1970). As the static yield stress was adopted, the numerical values of W_f/H are a little larger than the experimental ones owing to the strain rate sensitivity effect of mild steel material.

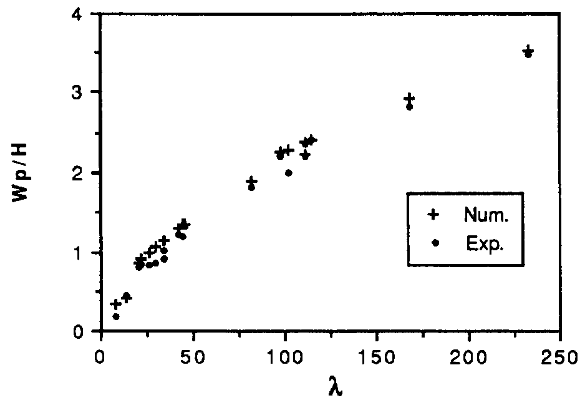
4.2. *Transient deformation mode of square plate under pressure pulse*

Numerical analyses were performed for one of the tests on an aluminium alloy plate presented in Section 3 to reproduce the transient response under measured time history of pressure (Fig. 8). The peak pressure is the mean value of a series test under identical experimental conditions.

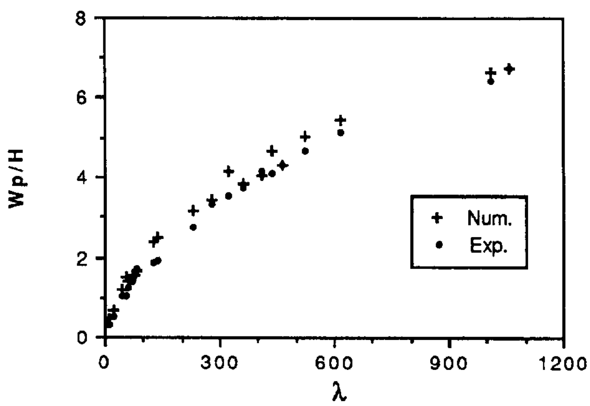
The calculated central deflection history of the square plate is given in Fig. 9. For a given pressure pulse, it takes some time to speed up the plate. When the plate reaches its maximum velocity, it will move at this speed for a while and then drops quickly to zero to end the motion. The calculated maximum permanent deflection is 35.65 mm which is reached at 340.5 μs. A comparison between the calculated (Fig. 9) and experimental deflection history near the plate centre (Fig. 5a) indicates a good agreement.

The transient deformation profiles were calculated by the numerical program (Fig. 10). For the purpose of easy comparison, the transient deformations are plotted at the same time instant as the experimental ones shown in Fig. 4. The numerical program simulated transient response satisfactorily. The numerical results presented a full picture of transient deformation profiles right up to the boundary, although this region is not covered experimentally. It is noted that under pressure pulse loading, the maximum slope of the plate near the clamping edge is not achieved at the early stage of the deformation process.

To visualize the plate configuration during the deformation process of the aluminium alloy plate, the 3-D deformation profile from the numerical program are presented in Fig. 11 at successive intervals. It is clear from Fig. 11 that the hinge lines of square flat plateau which initiated at boundaries contract in length with time and finally a pyramid-like permanent deformation profile is formed. In the numerical simulation, the material elasticity was retained. Nevertheless, when the plate is subjected to intense dynamic loading, the



(a) Aluminium alloy plate (19 specimens)



(b) Mild steel plate (22 specimens)

Fig. 7. Comparison between test and numerical results of maximum permanent deflection [test data of Jones *et al.* (1970)]. (a) Aluminium alloy rectangular plate, (b) mild steel rectangular plate.

elastic effect becomes less important and it is reasonable to correlate the edge line with the moving plastic hinge line used in the Rigid Plastic Analysis.

4.3. *Transient deformation mode of rectangular plate under an impulse*

The transient response of the aluminium alloy rectangular plate used in test No. 19 by Jones *et al.* (1970) is also simulated by the present numerical program. The plate was subjected to an impulse which produced an initial velocity of 119 m s^{-1}.

The calculated maximum permanent deflection is 10.83 mm and measured value is 10.79 mm. The central portion of the plate travels at its given velocity for approximately 90 μs, and slows down to reach its maximum deflection at 110 μs (Fig. 12).

The predicted transient deformation profiles in Figs 13 and 14 indicate that the plate reaches its maximum slope at clamping edges at a quite early stage of the deformation process. This is different from the case examined earlier for pressure pulse loading. The hinge line of the rectangular flat plateau travelling from the long edge of the plate reaches the symmetry line first. There is not enough kinetic energy for the hinge line from the short

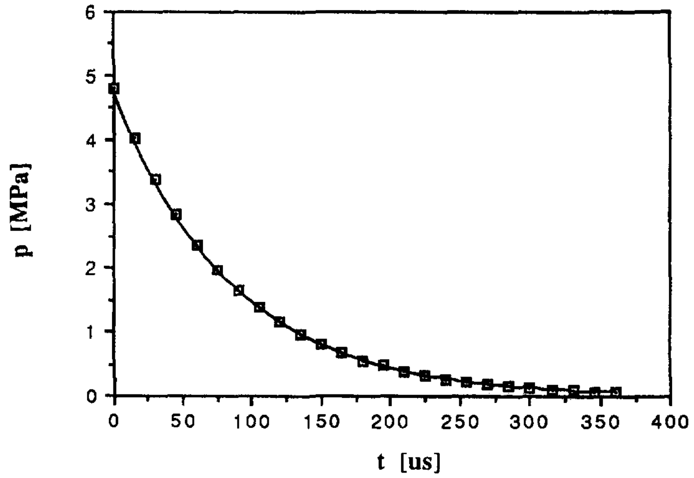


Fig. 8. Applied pressure-time history.

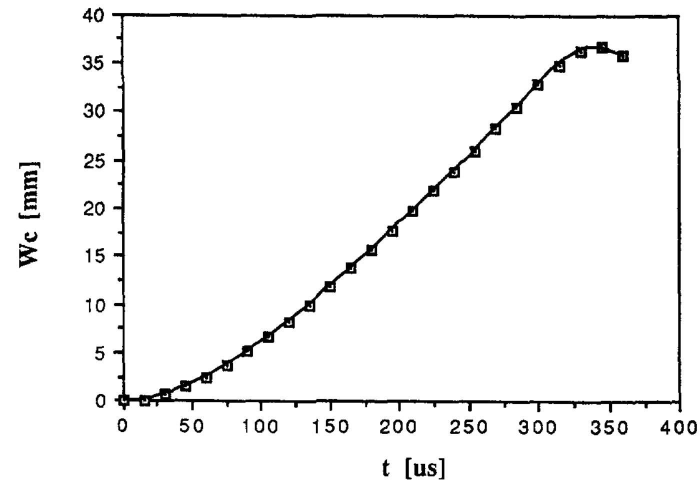


Fig. 9. Deflection–time history at centre of aluminium alloy square plate under charge explosive loading (numerical method).

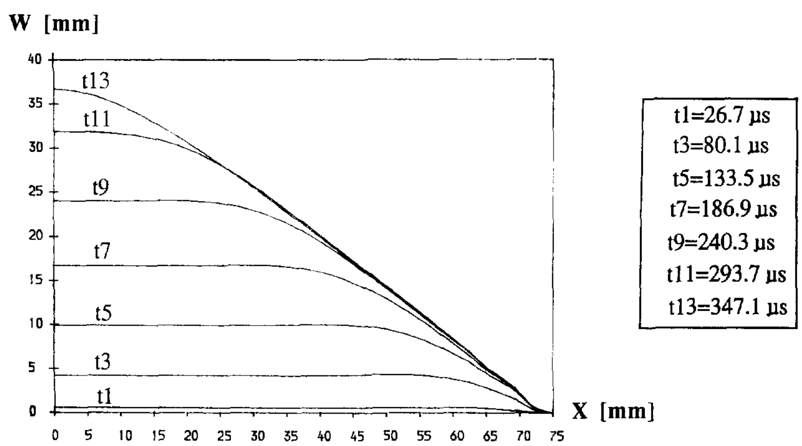


Fig. 10. Transient deformation profiles of aluminium alloy square plate (numerical method).

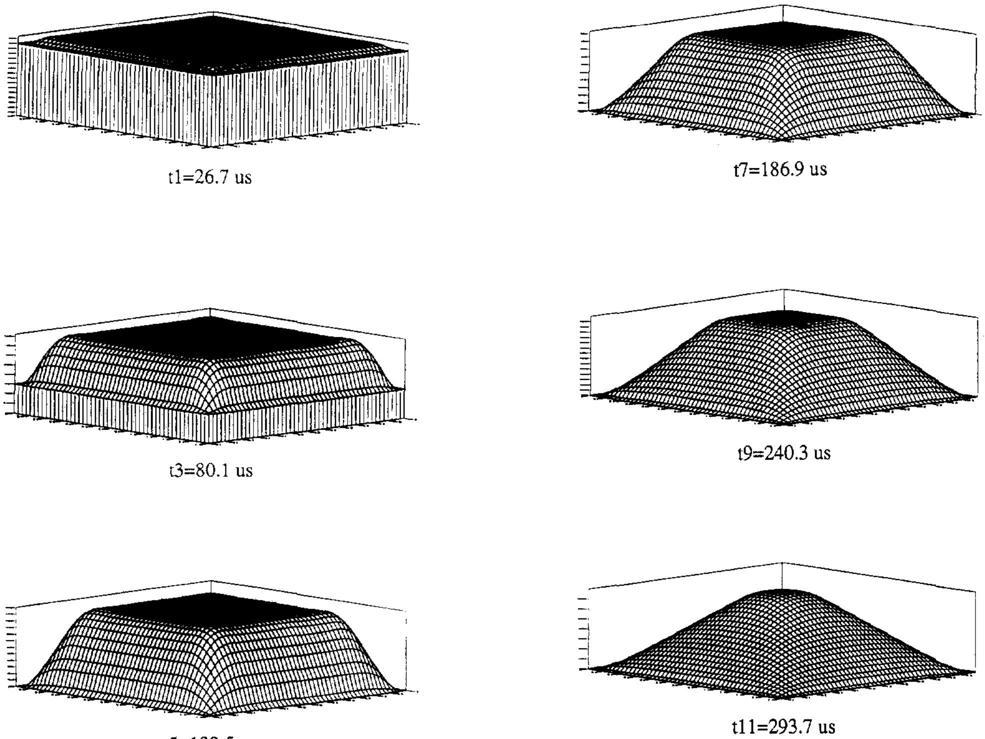


Fig. 11. Deformation process of aluminium alloy square plate (numerical method).

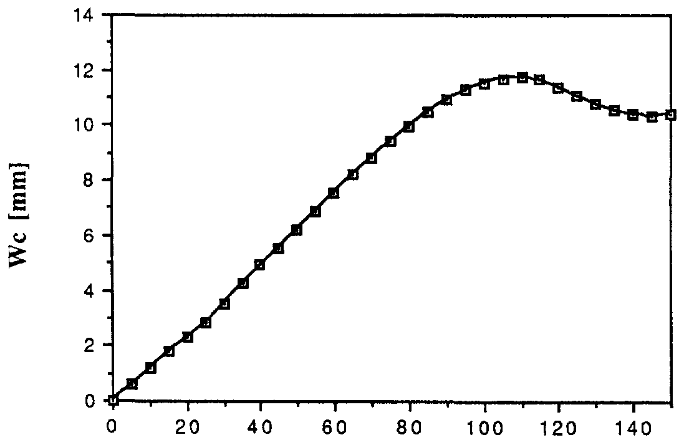


Fig. 12. Deflection time history at centre of aluminium alloy rectangular plate under sheet explosive loading (numerical method).

clamping edge to reach the plate centre and a permanent set with an envelope shape is consequently formed.

5. CONCLUDING REMARKS

The transient mode of a square plate under an explosive pressure pulse had been obtained by using a new optical technique. Two typical experimental cases have been given here to demonstrate the transient deformation mode of square plates made from aluminium alloy and mild steel materials. This optical technique proved to be reliable and accurate in obtaining the transient deformation mode of a square plate under explosive loading. The

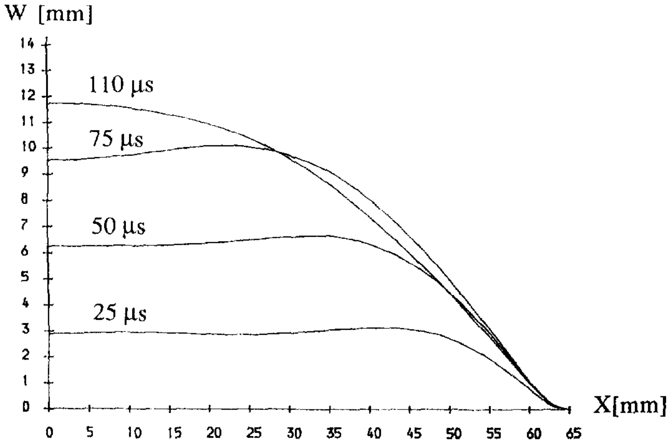


Fig. 13. Transient deformation profiles of aluminium alloy rectangular plate along X-axis (numerical method).

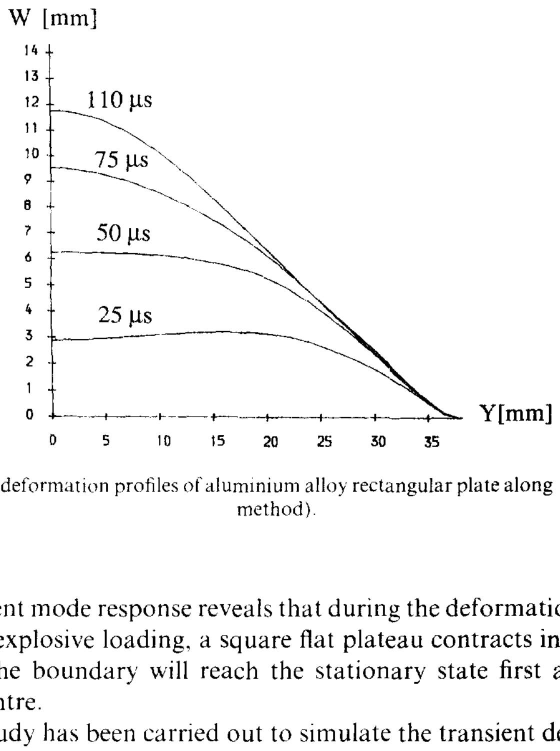


Fig. 14. Transient deformation profiles of aluminium alloy rectangular plate along Y-axis (numerical method).

experimental transient mode response reveals that during the deformation process of square plate under charge explosive loading, a square flat plateau contracts in size with time, and the material near the boundary will reach the stationary state first and then propagate toward the plate centre.

A numerical study has been carried out to simulate the transient deformation mode of aluminium alloy plate under an explosive pressure pulse, which was examined in the experiment. The effects of finite deflection, material elasticity and strain hardening were included in the formulation. Numerical prediction showed good agreement with the experimental results not only of permanent deflection but also of transient deformation profiles.

The transient mode of the rectangular plate under impulsive loading has also been examined numerically. It transpires from the numerical analysis that the deflection–time curve at the plate centre and the time for the plate to reach its maximum slope at the clamping edge varies with type of loading.

Combined experimental and numerical investigations provide valuable information for the understanding of transient behaviour of a thin plate under explosive loading and for the development of approximate procedure using the Rigid Plastic Method.

Acknowledgement—The author is grateful to the late Dr G. M. Hamilton of Reading University for his comments on the first draft of the manuscript.

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